Doubly integrals, simple example

\[ \iint_R f(x,y) \, dA = \int_a^b \int_{y_1(x)}^{y_2(x)} f(x,y) \, dy \, dx \]

\[ \int_0^1 \int_0^{x^2} xy \, dy \, dx = \]

\[ = \int_0^1 \left[ \frac{1}{2} x^2 y^2 \right]_0^{x^2} \, dx = \]

\[ \int_0^1 \frac{1}{2} x^5 \, dx = \left[ \frac{1}{2} \cdot \frac{1}{6} x^6 \right]_0^1 = \frac{1}{12} \]
Double Integrals 

\[ \int_{a}^{b} \int_{y_1(x)}^{y_2(x)} f(x, y) \, dy \, dx \]

For the region \( R \) bounded by \( y = x^2 \), \( y = 1 \), \( x = 0 \), and \( x = 1 \):

\[ \int_{0}^{1} \int_{0}^{1} xy \, dy \, dx \]

\[ \int_{0}^{1} \left[ y \cdot \frac{1}{2}y^2 \right]_{y=0}^{y=x^2} \, dx = \int_{0}^{1} \left[ \frac{1}{2}x^3 \right] \, dx = \left[ \frac{1}{8}x^4 \right]_{0}^{1} = \frac{1}{8} \]

\[ \int_{0}^{1} \frac{1}{2}x^5 \, dx = \left[ \frac{1}{2} \cdot \frac{1}{6}x^6 \right]_{0}^{1} = \frac{1}{12} \]
\[
\int_0^1 \int_{x^4}^x f(x,y) \, dy \, dx = \int_0^1 \left[ y \cdot (x-1) \right]_{y=x^4}^{y=x} \, dx \\
= \int_0^1 x \cdot (x-1) - x^4 (x-1) \, dx \\
= \int_0^1 x^2 - x - x^5 + x^4 \, dx
\]